THE PASSAGE OF INTERNAL WAVES FROM GENERATION TO PROPAGATION

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Abstract. Internal waves are ubiquitous in density stratified water bodies. Once generated, internal waves play an integral role in the transfer of energy and momentum throughout the environment until finally the waves are dissipated, often in quite localized regions. The focus here is on internal waves in semi-enclosed water bodies, such as the coastal ocean, where the forcing is predominantly due to the barotropic tide. We discuss recent work on the generation and propagation of waves using a combination of laboratory experiments, numerical simulations and field observations in the coastal ocean from the Australian North West Shelf. We examine the generation process due to the interaction between the barotropic tide and the bottom topography and the subsequent evolution of waves as they propagate into shallower water.

1. Introduction

Internal waves transfer energy and momentum throughout the stratified interior of natural water bodies. In smaller water bodies such as lakes, internal waves result primarily from wind forcing and the confined nature of the water bodies can often result in resonance effects (Wake et al [1]; Boegman and Ivey [2]) and complex internal wave fields. In coastal regions such as the Australian North West Shelf (NWS), internal waves result primarily from the interaction between the barotropic forcing of stratified water over the bottom topography, resulting in a highly spatially variable and time-dependent internal wave field. Energy is transferred from larger to smaller scale waves and can lead to highly non-linear internal waves being formed with associated strong local velocity fields which can be of considerable engineering and environmental importance.

Internal waves generated by tides, or internal tides, have long been of interest because of their importance in the coastal ocean, and theory and observations have been recently reviewed by Vlasenko et al [3]. The generation of the internal waves by the tide is dependent upon three factors: the form of the density stratification, the shape of the bottom topography, and the intensity of the barotropic forcing (e.g. Griffiths and Grimshaw [4]). Early work on the generation process has built on the knowledge from hydraulic theory and used an internal Froude number to describe the flow (e.g. Lansing and Maxworthy [5]), and here we extend this approach and describe some laboratory experiments and field observations which examine both the generation and propagation process.

2. Laboratory experiments

Figure 1 below shows the configuration of the experiments. A two-dimensional continental shelf/slope configuration was installed on the bottom of a tank filled
with either a two-layer or linear density stratification. Here we only consider the two-layer case. A tide was then generated offshore by vertically oscillating a triangular shaped piston producing a barotropic velocity at the base of the slope of the form $U_0 \sin \omega t$, where $\omega$ is the tidal frequency. The resulting two-dimensional (2D) density field was visualized with a Light Attenuation (LA) technique, and 2D velocity fields were recorded with a combination of an Acoustic Doppler Velocimeter and Particle Tracking Velocimetry (PTV). For these experiments we varied two parameters, the amplitude of the barotropic forcing $U_0$ and the depths of the two-layer density stratification, and a wide variety of responses were observed.

3. Laboratory Results

Figure 2 shows an example of internal wave response by tidal forcing of density stratification over topography. During the ebb phase of the flow (Fig 2a), the pycnocline forms a long wave of depression over the slope. As the tide turns, this wave of depression propagates onshore, but the onshore wave goes through a turning point and in this example, the shoreward propagating wave steepens to form a bore inshore of the shelf break. As it propagates inshore the bore subsequently evolves into a non-linear wave (Fig 2b) packet and continues
to propagate inshore even as the barotropic tide turns. If the lower layer on the shelf is equal to or larger in thickness than the upper layer, the character of the propagating wave remains linear throughout (Fig 3 a,b). At the other extreme, if the lower layer is below the shelf break, then there is little internal tide response observed on the shelf.

It is convenient to separate the dynamics into two phases: a generation phase and a propagation phase. Consider first the generation process during the ebb tidal phase where the significant dynamics occur near the shelf break, as shown in Figures 2 and 3, for example,. Due to the combination of the local bottom slope $S$ and the barotropic velocity $U_0$ at the shelf break, the near bottom induced vertical velocity will be $w = SU_0$. Assuming mixing effects are negligible, then close to the shelf break the induced depression of the overlying pycnocline $\delta$ is

$$\delta \sim S \frac{U_0}{\omega}$$

where we assume the stratification is strong enough that $\delta$ is small compared to the water depth. Defining the equivalent layer depth for the two-layer density stratification on the shelf as $h_E = h_1 h_2 S / (h_1 + h_2 S)$, then (Lim et al. [6])

$$\frac{\delta}{h_E} \frac{U_0 S}{\omega h_E} = R_G$$

where $R_G$ is defined as the baroclinic generation parameter. It is also useful to know the Froude number $Fr_0 = U_0 / \sqrt{g h_E}$, the ratio of the barotropic velocity at the shelf break to the internal wave speed.

In the general case where the pycnocline lies anywhere between the surface and the shelf break, the layer depth ratio
\[ \beta = h_1/(h_1 + h_{2S}) \]  

(3)

also governs the response and as \( \beta \to 0, \delta \to 0 \). As required, the two independent parameters \( R_G \) and \( \beta \) contain a measure of the form of the density stratification, the shape of the bottom topography, and the intensity of the barotropic forcing. For fixed \( \beta \), for \( R_G \to 0 \) there will be only a weak response and dissipation will eventually damp out any wave formation. Similarly, for fixed \( R_G \), for \( \beta \to 0 \) there will be only a weak response. Thus, both parameters in (2) and (3) govern the strength of internal tide generation.

Consider now the propagation away from the generation region over the slope. The character will be strongly dependent on the ratio of \( \delta \) to the lower layer depth \( h_{2S} \) on the shelf, that is

\[ \frac{\delta}{h_{2S}} = R_G \beta \]  

(4)

Clearly if \( (\delta/h_{2S}) \geq 1 \) we would expect to see highly non-linear waves to evolve on the shelf as in the example shown in Figure 2b. Thus again the response is governed by the two parameters \( R_G \) and \( \beta \). The parameters in (2) and (3) provide a simple description based on external parameters of the character of both the process of internal tide generation and propagation, and the results for a series of laboratory runs are shown in the Table below.

<table>
<thead>
<tr>
<th>( \beta )</th>
<th>( F_{10} )</th>
<th>( R_G )</th>
<th>Wave Response</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Lab</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.33</td>
<td>0.22</td>
<td>0.36</td>
<td>Linear</td>
</tr>
<tr>
<td>0.33</td>
<td>0.33</td>
<td>0.37</td>
<td>Linear</td>
</tr>
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<td>0.33</td>
<td>0.60</td>
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<td>Linear</td>
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<td>0.98</td>
<td>0.47</td>
<td>Linear + mixing</td>
</tr>
<tr>
<td>0.67</td>
<td>0.21</td>
<td>0.36</td>
<td>Non-linear</td>
</tr>
<tr>
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<td>0.33</td>
<td>0.37</td>
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<tr>
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<td>Non-linear</td>
</tr>
<tr>
<td>0.67</td>
<td>0.95</td>
<td>0.47</td>
<td>Non-linear + Mixing</td>
</tr>
</tbody>
</table>

| **Field** | | | |
| 0.68 | 0.21 | 0.65 | Non-linear |
| 0.72 | 0.71 | 1.27 | Non-linear |
4. Field Observations and Discussion

The North West Shelf of Western Australia has some of the largest internal tides in the coastal ocean and this is currently being investigated using combined field observation program (Van Gastel et al [7]) and a hydrid numerical modeling scheme (Meuleners et al [8]), consisting of a nesting of the three models: BLUElk, ROMS and SUNTANS, with the inner model SUNTANS being a fully non-hydostatic three dimensional unstructured grid model (Fringer et al [9]). During late summer, a strong two layer stratification forms on the Shelf with a thick surface mixing layer and a thin lower layer. Two examples are shown in Figure 4, during firstly a spring tide and secondly a neap tide, and large amplitude internal waves of elevation are observed at the measurement location in both cases.

The details of the dynamics are discussed elsewhere [5,6], but the parameters for each of these two cases are shown in the table above. Barotropic tidal currents were taken from [6] at the shelf break, defined to be at the 150 m isobath at this location. Comparison of the lab and field examples shown in the table suggests the simple scaling arguments above can be used to separate the flow into three basic regimes. If the forcing is very intense with $R_G > 0.4$, hence a local Richardson number is small, then mixing starts to occur at the generation site disrupting the transfer of energy from the barotropic to baroclinic mode. If $R_G < 0.4$ and $\beta < 0.5$, then linear baroclinic waves emerge from the generation zone. If $R_G < 0.4$ and $\beta > 0.5$, then non-linear
baroclinic waves can emerge from the generation zone. In the limit when $R_G \rightarrow 0$ there is insufficient energy available to generate any baroclinic wave.

Acknowledgments
Thanks to Kenny Lim, Paul van Gastel and Mike Meuleners for comments, and to the Australian Research Council and Woodside Energy Ltd. for financial support.

References
2. L. Boegman and G.N. Ivey. 5th ISEH Conference (2007), Submitted